Radiation damping in real time

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We study the nonequilibrium dynamics of a charge interacting with its own radiation, which originates the radiation damping. The *real-time* equation of motion for the charge and the associated Langevin equation is found in classical limit. The equation of motion for the charge allows one to obtain the frequency-dependent coefficient of friction. In the lowest order we find that although the coefficient of static friction vanishes, there is dynamical dissipation represented by a non-Markovian dissipative kernel.

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I. INTRODUCTION AND MOTIVATION

The quantum dynamics of a dissipative system is still an open problem in physics, where the standard quantization scheme, that is based on the existence of either a Hamiltonian or a Lagrangian function for the system in which we are interested, is not applicable. It is well known that we cannot obtain an equation of motion from the application of the classical Lagrange's or Hamilton's equations to any Lagrangian or Hamiltonian that has explicit time dependence.

Among some approaches to study this problem, there is one, proposed by Feynman and Vernon [1] that gives a very good result. Such approach can be used to study questions related to quantum dissipative systems [2], phase transition [3], and in particular to the quantum dynamics of a Brownian particle [4]. Another important system with appropriated characteristic that allows one to use this approach is the study of quantum dynamics of accelerated charge. It is a dissipative system once an accelerated charge loses energy, linear momentum, and angular momentum carried by the radiation field. The effect of these losses to the motion of charge is known as radiation damping.

Some works have been done about the motion of a charged particle under the effect of radiation damping. Just to cite some, in the classical limit we have the ones due to Becker [5] and Lorentz [6] and in the relativistic limit the work by Dirac [7]. Another derivation of the radiation damping force is discussed by Hartemann and Luhmann (HL) [8]. In that paper, HL obtained classically a covariant expression for the instantaneous radiation damping force on an accelerated charged particle. This solution had eluded all authors before them, including those in Refs. [5,6,9–12]. HL derived the Abraham-Becker radiation damping force classically for the first time. In this work we attempt to derive the same radiation damping using quantum dynamics, getting the formal solution of the equivalent quantum case that is in accordance with the results from HL, where the force is linear in the particle velocity.

In another paper, Barone and Caldeira [13] obtained a nonrelativistic quantum formulation for an accelerated electron. They made use of particle plus reservoir model, ''integrating out'' the degrees of freedom of fields leading to an effective action for the electron. They applied that formulation to the problem of electronic interference.

In this paper we study the nonequilibrium dynamics of radiation damping phenomenon, using the Schwinger-Keldysh [9,14–18,10,19–21] formulation of nonequilibrium statistical mechanics to obtain the real-time equations of motion for the charged particle. The choice in using the real-time field theory instead of the imaginary-time (Euclidean) formalism, named after Matsubara, is the fact that, in Fourier language, the Matsubara formalism involves discrete complex energies, which appear both on internal and external lines of Feynman diagrams. The internal energies have to be summed over. The external energies, on the other hand, pose a problem because they define the Green functions at a discrete set of points in the complex energy plane. But in order to answer dynamical questions, a knowledge of real-time Green functions is usually indispensable. This implies that the temperature Green functions have to be extended from the discrete energies to the real axis. This extension can be obtained by a process of analytic continuation, but in the case of several complex variables this is a mathematically difficult task. Even in the case of a single external energy, one is confronted with the problem that such an analytic extension is not unique without further delimitations. So the Matsubara formalism is well suited to the evaluation of static thermodynamic properties and the basic disadvantage of the Matsubara formalism lies in the unphysical representation of time and energy. Analytic continuation can be avoided by a different approach called real-time field theory, based on the concept of a closed time contour in the complex plane running parallel to the real-time axis and back. It involves the use of both time- and antitime-ordered Green functions and gives rise to an effective doubling of the degrees of freedom. In the real-time formalism, the Green functions can be directly obtained as functions of continuous real energy variables.

Using the real-time formalism, radiation field is treated in the coherent states, since we are interested in obtaining the equation of motion for the particle in the classical limit. This is achieved making a partial trace over the radiation field getting the effective action for the particle and the corresponding Langevin equation.

The Langevin equation leads us to identify the dissipative kernels and the noise correlation function that reflects the interaction of the charge with the radiation field. These two terms are related by the fluctuation-dissipation theorem [22].

This paper is organized as follows. In Sec. II we write down the Hamiltonian of the system charge plus radiation field, quantizing the field in terms of creation and annihilation operators in the conventional way. In Sec. III, we integrate out the radiation field degrees of freedom, obtaining the effective action, the equation of motion in real time, and the Langevin equation for the particle. We present our conclusions in Sec. IV pointing out directions of future research. In the Appendix, we present the technical details leading to the generating functional in *real time*.

II. THE HAMILTONIAN OF THE SYSTEM

The Hamiltonian describing a charge interacting with its own radiation field can be written as

$$H=H_e+H_f \tag{2.1}$$

where

$$H_e=\frac{1}{2m}(\vec{p}-e\vec{A})^2 \tag{2.2}$$

is the Hamiltonian of a moving charge e and mass m in the presence of electromagnetic field [23].

$$H_f=\frac{1}{2}\int d^3\vec{x}(\vec{E}^2+\vec{B}^2) \tag{2.3}$$

is the the Hamiltonian of the electromagnetic field [11].

We will use the gauge $\vec{\nabla}\cdot\vec{A}=0$ and $\phi=0$ once we suppose the fields interacting with the charge was created by the charge in previous time, so when the charge interacts with them, the fields are free. As free fields, they are transverse fields, so the above conditions hold.

Expanding $\vec{A}$ and its conjugated momentum $\vec{P}$ in terms of normal modes [12] we get

$$\vec{A}(\vec{x},t)=\sum_{k\lambda} q_{k\lambda}(t)\vec{u}_{k\lambda}(\vec{x}), \tag{2.4}$$

$$\vec{P}(\vec{x},t)=\sum_{k\lambda} p_{k\lambda}(t)\vec{u}_{k\lambda}(\vec{x}), \tag{2.5}$$

where

$$\vec{u}_{k\lambda}(\vec{x})=L^{-3/2}\hat{\epsilon}_{k\lambda}\exp(i\vec{k}\cdot\vec{x}), \tag{2.6}$$

that are plane waves in a box with volume L^3 and $\hat{\epsilon}_{k\lambda}$ ($\lambda=1,2$) are the polarization vectors satisfying the conditions $\vec{k}\cdot\hat{\epsilon}_{k\lambda}=0$, $\hat{\epsilon}_{k\lambda}\cdot\hat{\epsilon}_{k\lambda'}=\delta_{\lambda\lambda'}$. $\vec{u}_{k\lambda}(\vec{x})$ are normalized as

$$\int \vec{u}^{*}_{k\lambda}(\vec{x})\cdot\vec{u}_{k'\lambda'}(\vec{x})d^3\vec{x}=\delta_{kk'}\delta_{\lambda\lambda'}. \tag{2.7}$$

Substituting Eqs. (2.4) and (2.5) into the charge and field Hamiltonians we get [12]

$$H_e=\frac{1}{2m}\left[\vec{p}-e\sum_{k\lambda} q_{k\lambda}(t)\vec{u}_{k\lambda}(\vec{x})\right]^2, \tag{2.8}$$

$$H_f=\frac{1}{2}\sum_{k\lambda}[p^2_{k\lambda}(t)+k^2q^2_{k\lambda}(t)], \tag{2.9}$$

which have the form expected for the behavior of the radiation field considered as a set of infinity uncoupled harmonic oscillators.

For quantizing this system, we make use of creation and annihilation operators, such that

$$q_{k\lambda}(t)=\sqrt{\frac{\hbar}{2k}}[a^{\dagger}_{k\lambda}(t)+a_{k\lambda}(t)], \tag{2.10}$$

$$p_{k\lambda}(t)=i\sqrt{\frac{\hbar k}{2}}[a^{\dagger}_{k\lambda}(t)-a_{k\lambda}(t)], \tag{2.11}$$

satisfying the usual commutation relation,

$$[q_{k\lambda}(t),p^{\dagger}_{k'\lambda'}(t)]=i\hbar\,\delta_{kk'}\delta_{\lambda\lambda'} \tag{2.12}$$

with $q^{\dagger}_{k\lambda}(t)=q_{-k\lambda}(t)$ and $p^{\dagger}_{k\lambda}(t)=p_{-k\lambda}(t)$ since the vector potential and its conjugate momentum are taken to be reals.

Substituting Eqs. (2.10) and (2.11) in the equations (2.8) and (2.9), we get

$$H=\frac{1}{2m}(\vec{p}-h[a^{\dagger}_{k\lambda},a_{k\lambda}])^2+H_f[a^{\dagger}_{k\lambda},a_{k\lambda}], \tag{2.13}$$

where

$$h[a^{\dagger}_{k\lambda},a_{k\lambda}]=e\sum_{k\lambda}\sqrt{\frac{\hbar}{2k}}[a^{\dagger}_{k\lambda}(t)+a_{k\lambda}(t)]\vec{u}_{k\lambda}(\vec{x}), \tag{2.14}$$

$$H_f[a^{\dagger}_{k\lambda},a_{k\lambda}]=\hbar\sum_{k\lambda}\omega_k(a^{\dagger}_{k\lambda}(t)a_{k\lambda}(t)+1/2). \tag{2.15}$$

As mentioned before, our main goal in this paper is to study the dynamics of a charge interacting with its own radiation field. Physically, we are interested in the correlation function of the charge and not in the properties of the field. So, we will apply the formalism of path integral in real time, considering the photons coming from the radiation as a bath that will be traced over to give us the effective action for the charge.

As radiation field appears only when the charge is accelerated, we introduce an external classical field, under which the charge has an accelerated motion. This field couples to the charge as $e\vec{\mathcal{E}}(t)$ where we can define $\vec{j}(t)$ as $\vec{j}(t)=e\vec{\mathcal{E}}(t)$, giving the following contribution to the Lagrangian:

$$\mathcal{L}_{ext}=-\vec{j}(t)\cdot\vec{x}(t), \tag{2.16}$$

where we suppose the field is switched on at $t=0$,

$$\vec{j}(t)=\begin{cases}\vec{0} & \text{if} \quad t<0,\\ \vec{j} & \text{if} \quad t>0,\end{cases} \tag{2.17}$$

giving us the Lagrangian,

$$\mathcal{L}[\dot{\vec{x}},a^{\dagger}_{k\lambda},a_{k\lambda}]=\frac{m}{2}\dot{\vec{x}}^2+\dot{\vec{x}}e\sum_{k\lambda}\sqrt{\frac{\hbar}{2k}}[a^{\dagger}_{k\lambda}(t)+a_{k\lambda}(t)]\vec{u}_{k\lambda}(\vec{x})-\hbar\sum_{k\lambda}\omega_k(a^{\dagger}_{k\lambda}(t)a_{k\lambda}(t)+1/2)-\vec{j}(t)\cdot\vec{x}(t). \tag{2.18}$$

III. THE REAL-TIME GENERATING FUNCTIONAL

There are different ways to fix the initial condition for this kind of problem, such as the one where the system in interest, in our case the particle, and the reservoir (the radiation field) are interacting since the beginning [4]. However, we will use one, where the total density matrix for the particle field is in thermal equilibrium at a temperature T until the particle decouples at the initial time $t=0$ [1,24]. Here, the interaction is switched on when the charge is put in the presence of the classical field, because it is when the particle starts to radiate [1,24] and the system departs from the condition where the particle and the bath are in equilibrium [25–28].

At time zero we write the total density matrix as a product of the density matrices for the free charge and its own radiation field (the bath)

$$\rho(t_i)=\rho_e(t_i)\otimes\rho_f(t_i), \tag{3.1}$$

where $\rho_e(t_i)$ is the density matrix for the particle, considering it as a free particle, $\rho_e(t_i)=|\vec{x}\rangle\langle\vec{x}|$ and $\rho_f(t_i)$ is the density matrix for the bath of photons in thermal equilibrium at temperature T, given by

$$\rho_f(t_i)=\frac{e^{-\beta H_f}}{\mathbb{Z}}, \tag{3.2}$$

with

$$\mathbb{Z}=\mathrm{tr}_f(e^{-\beta H_f}). \tag{3.3}$$

H_f is the Hamiltonian for the free field given by Eq. (2.15).

The closure relation of the system charge+radiation field is

$$\int\frac{d^2\alpha}{\pi^N}\int d\vec{x}|\vec{x},\alpha\rangle\langle\vec{x},\alpha|=1, \tag{3.4}$$

where we use the coordinate representation for the particle

$$\vec{x}|\vec{x}\rangle=\vec{x}|\vec{x}\rangle \tag{3.5}$$

and the coherent states representation for the field radiated by the particle

$$|\alpha\rangle=|\alpha_1\rangle\otimes|\alpha_2\rangle\otimes\cdots\otimes|\alpha_N\rangle \tag{3.6}$$

with N being infinity.

In terms of evolution operator $U(t,-\infty)$ the generating functional in real time is given by

$$Z[j]=\frac{1}{\mathrm{tr}\,\rho(t_i)}\int\frac{d^2\alpha}{\pi^N}\langle\alpha,\vec{x}|U_{j^+}(t,-\infty)\rho(t_i)U^{-1}_{j^-}(t,-\infty)|\vec{y},\alpha\rangle \tag{3.7}$$

or

$$Z[j]=\frac{1}{\mathrm{tr}\,\rho(t_i)}\int d\vec{x}'\int d\vec{y}'\rho_e(\vec{x}',\vec{y}',t_i)\times\int\frac{d^2\alpha}{\pi^N}\frac{d^2\gamma}{\pi^N}\frac{d^2\delta}{\pi^N}\langle\gamma|\rho_f(t_i)|\delta\rangle\times\langle\vec{x},\alpha|U_{j^+}(t,-\infty)|\vec{x}',\gamma\rangle\langle\vec{y}',\delta|U^{-1}_{j^-}(t,-\infty)|\vec{y},\alpha\rangle \tag{3.8}$$

with

$$\rho_e(\vec{x}',\vec{y}';t_i)=\langle\vec{x}'|\rho_e(t_i)|\vec{y}'\rangle, \tag{3.9}$$

$$\langle\gamma|\rho_f(t_i)|\delta\rangle=\frac{1}{\mathbb{Z}}\prod_{k=1}^{N}\exp\left\{-\frac{|\gamma_k|^2}{2}-\frac{|\delta_k|^2}{2}+\gamma_k^*\delta_k e^{-\beta\hbar\omega_k}\right\}, \tag{3.10}$$

$$\mathbb{Z}(t)=\prod_{k=1}^{N}[1-\exp(-\beta\hbar\omega_k)]^{-1}e^{\beta\hbar\omega_k/2}. \tag{3.11}$$

Nonequilibrium Green's functions are obtained as functional derivatives with respect to the sources j and J (see the Appendix). There are four types of free propagators [9,14–18,10,19–21]. Using them, we get

$$Z[j,J]=\frac{1}{\mathrm{tr}\,\rho(t_i)}\int d\vec{x}'d\vec{y}'\rho_e(\vec{x}',\vec{y}',t_i)\times\int\mathcal{D}\vec{x}^+(\tau)\mathcal{D}\vec{x}^-(\tau)\times\exp\left\{\frac{i}{\hbar}(S_0[\vec{x}^+]-S_0[\vec{x}^-])\right\}\mathcal{Z}[J^+,J^-], \tag{3.12}$$

where

$$\mathcal{D}\vec{x}(\tau)=\lim_{N\to\infty\ \varepsilon\to0}\left\{\left(\frac{m}{2\pi i\hbar}\right)^{N/2}\prod_{j=1}^{N-1}d\vec{x}_j\right\} \tag{3.13}$$

and

$$S_0[\vec{x}]=\int_{-\infty}^{t}d\tau\left\{\frac{m}{2}\dot{\vec{x}}^2(\tau)-\vec{j}(\tau)\vec{x}(\tau)\right\}. \tag{3.14}$$

$S_0[\vec{x}]$ is the action for the charge when it is not coupled to the radiation field.

The term $\mathcal{Z}[J^+,J^-]$ in Eq. (3.12) plays the role of influence functional [24] equivalent to the harmonic oscillator generating functional [9,14–18,10,19,20],

$$\mathcal{Z}[J^+,J^-]=\exp\Bigg\{-\sum_{k\lambda}\sum_{\lambda'}\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'[J_{k\lambda}^{+\dagger}(\tau)J_{k\lambda'}^{+}(\tau')\times\{\theta(\tau-\tau')+\eta_k\}+J_{k\lambda}^{-\dagger}(\tau)J_{k\lambda'}^{-}(\tau')\{\theta(\tau'-\tau)+\eta_k\}-J_{k\lambda}^{+\dagger}(\tau)J_{k\lambda'}^{-}(\tau')\eta_k-J_{k\lambda}^{-\dagger}(\tau)J_{k\lambda'}^{+}(\tau')\times(1+\eta_k)]e^{-i\omega_k(\tau-\tau')}\Bigg\}, \tag{3.15}$$

where the linear coupling acts as a ''source'' term and may be written as

$$J_{k\lambda}(t)=J_{k\lambda}^{\dagger}(t)=-\frac{e}{\sqrt{2\hbar k}}\dot{\vec{x}}(t)\cdot\vec{u}_{k\lambda}(\vec{x}). \tag{3.16}$$

So far, no approximation has been made, that is, our result for the generating functional (3.15) is exact.

Writing $\mathcal{Z}[J^+,J^-]$ in terms of the particle coordinates, we get the influence functional

$$\mathcal{F}[\vec{x}^+,\vec{x}^-]=\exp\Bigg\{-\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'\sum_{k\lambda}\sum_{\lambda'}\frac{e^2}{2\hbar kL^3}[\{\dot{\vec{x}}^+(\tau)\cdot\hat{\epsilon}_{k\lambda}\}e^{i\vec{k}\cdot\vec{x}^+(\tau)}\{\theta(\tau-\tau')+\eta_k\}\{\dot{\vec{x}}^+(\tau')\hat{\epsilon}_{k\lambda'}\}e^{i\vec{k}\cdot\vec{x}^+(\tau')}$$
$$-\{\dot{\vec{x}}^-(\tau)\cdot\hat{\epsilon}_{k\lambda}\}e^{i\vec{k}\cdot\vec{x}^-(\tau)}\{\theta(\tau'-\tau)+\eta_k\}\{\dot{\vec{x}}^-(\tau')\cdot\hat{\epsilon}_{k\lambda'}\}e^{i\vec{k}\cdot\vec{x}^-(\tau')}-\{\dot{\vec{x}}^-(\tau)\hat{\epsilon}_{k\lambda}\}e^{i\vec{k}\cdot\vec{x}^-(\tau)}(1+\eta_k)$$
$$\times\{\dot{\vec{x}}^+(\tau')\hat{\epsilon}_{k\lambda'}\}e^{i\vec{k}\cdot\vec{x}^+(\tau')}-\{\dot{\vec{x}}^+(\tau)\hat{\epsilon}_{k\lambda}\}e^{i\vec{k}\cdot\vec{x}^+(\tau)}(\eta_k)\{\dot{\vec{x}}^-(\tau')\hat{\epsilon}_{k\lambda'}\}e^{i\vec{k}\cdot\vec{x}^-(\tau')}]e^{-i\omega_k(\tau-\tau')}\Bigg\} \tag{3.17}$$

that bears all the information about the effect of the radiation field on the charge. When there is no interaction between the system of interest and environment the influence functional is equal to one.

This influence functional can be written as

$$\mathcal{F}[\vec{x}^+,\vec{x}^-]=\exp\Bigg\{-\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'\sum_{k}\frac{e^2}{\hbar kL^3}[\dot{\vec{x}}^+(\tau)\dot{\vec{x}}^+(\tau')\exp\{i\vec{k}\cdot\vec{x}^+(\tau)+i\vec{k}\cdot\vec{x}^+(\tau')G_k^{++}(\tau-\tau')$$
$$+\dot{\vec{x}}^-(\tau)\dot{\vec{x}}^-(\tau')\exp\{i\vec{k}\cdot\vec{x}^-(\tau)+i\vec{k}\cdot\vec{x}^-(\tau')\}G_k^{--}(\tau-\tau')+\dot{\vec{x}}^-(\tau)\dot{\vec{x}}^+(\tau')$$
$$\times\exp\{i\vec{k}\cdot\vec{x}^-(\tau)+i\vec{k}\cdot\vec{x}^+(\tau')\}G_k^{-+}(\tau-\tau')+\dot{\vec{x}}^+(\tau)\dot{\vec{x}}^-(\tau')\exp\{i\vec{k}\cdot\vec{x}^+(\tau)+i\vec{k}\cdot\vec{x}^-(\tau')\}G_k^{+-}(\tau-\tau'), \tag{3.18}$$

where

$$G^{++}(\vec{r}_1,t_1;\vec{r}_2,t_2)=G^{>}(\vec{r}_1,t_1;\vec{r}_2,t_2)\theta(t_1-t_2)+G^{<}(\vec{r}_1,t_1;\vec{r}_2,t_2)\theta(t_2-t_1),$$
$$G^{--}(\vec{r}_1,t_1;\vec{r}_2,t_2)=G^{>}(\vec{r}_1,t_1;\vec{r}_2,t_2)\theta(t_2-t_1)+G^{<}(\vec{r}_1,t_1;\vec{r}_2,t_2)\theta(t_1-t_2),$$
$$G^{+-}(\vec{r}_1,t_1;\vec{r}_2,t_2)=-G^{<}(\vec{r}_1,t_1;\vec{r}_2,t_2),$$
$$G^{-+}(\vec{r}_1,t_1;\vec{r}_2,t_2)=-G^{>}(\vec{r}_1,t_1;\vec{r}_2,t_2)=-G^{<}(\vec{r}_2,t_2;\vec{r}_1,t_1), \tag{3.19}$$

with

$$G_k^{<}(\tau-\tau')=\frac{i}{2\omega_k}[e^{i\omega_k(\tau-\tau')}(1+\eta_k)+e^{-i\omega_k(\tau-\tau')}\eta_k], \tag{3.20}$$

$$G_k^{>}(\tau-\tau')=\frac{i}{2\omega_k}[e^{-i\omega_k(\tau-\tau')}(1+\eta_k)+e^{i\omega_k(\tau-\tau')}\eta_k], \tag{3.21}$$

and $\eta_k=(e^{\beta\hbar\omega_k}-1)^{-1}$ is the ocupation number. So, the generating functional is

$$Z[j]=\frac{1}{\mathrm{tr}\,\rho(t_i)}\int d\vec{x}'d\vec{y}'\rho_e(\vec{x}',\vec{y}',t_i)\tilde{Z}[j], \tag{3.22}$$

where

$$\tilde{Z}[j]=\int\mathcal{D}\vec{x}^+(\tau)\mathcal{D}\vec{x}^-(\tau)\exp\{(i/\hbar)S[\vec{x}^+,\vec{x}^-]\} \tag{3.23}$$

with

$$S[\vec{x}^+,\vec{x}^-]=\int_{-\infty}^{t}d\tau\Bigg\{\frac{m}{2}[\dot{\vec{x}}^{+2}(\tau)-\dot{\vec{x}}^{-2}(\tau)]-\vec{j}(\tau)[\vec{x}^+(\tau)-\vec{x}^-(\tau)]+\int_{-\infty}^{t}d\tau'\sum_k\frac{e^2}{L^3}[\dot{\vec{x}}^a(\tau)\dot{\vec{x}}^b(\tau')\times\exp\{i\vec{k}\cdot[\vec{x}^a(\tau)+\vec{x}^b(\tau')]\}G_k^{ab}(\tau-\tau')]\Bigg\} \quad (3.24)$$

and $a,b=+,-$.

To study the dynamic of the charge, we need to find the equation of motion obeyed by the charge after we integrate out the radiation field. At this point we are not able to handle with the exact result above, so we need to use some approximation. The approximation we use is the background field approximation in which one fluctuation from the classic field is taken in account.

A. The equation of motion of charge and its properties

To use the background field approximation, we consider the situation where the exponential $e^{i\vec{k}\cdot\vec{x}}$ can be approximated as

$$e^{i\vec{k}\cdot\vec{x}^a(\tau)}\approx 1 \quad (3.25)$$

leading the influence functional (3.18) to

$$\mathcal{F}[\vec{x}^+,\vec{x}^-]=\exp\Bigg\{\frac{i}{\hbar}\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'\sum_k\frac{e^2}{L^3}[\dot{\vec{x}}^+(\tau)\dot{\vec{x}}^+(\tau')\times G_k^{++}(\tau-\tau')+\dot{\vec{x}}^+(\tau)\dot{\vec{x}}^-(\tau')G_k^{+-}(\tau-\tau')+\dot{\vec{x}}^-(\tau)\dot{\vec{x}}^-(\tau')G_k^{--}(\tau-\tau')+\dot{\vec{x}}^-(\tau)\dot{\vec{x}}^+(\tau')G_k^{-+}(\tau-\tau')]\Bigg\}. \quad (3.26)$$

In this limit, the equation of motion for the charge can be obtained in the background approximation, $\vec{x}^{\pm}(t)=\vec{q}(t)+\vec{\xi}^{\pm}(t)$, where $\langle\vec{\xi}^{\pm}(t)\rangle=0$ in all orders in perturbation theory. So the influence functional, $\mathcal{F}[\vec{x}^+,\vec{x}^-]$, reads

$$\mathcal{F}[\vec{q}+\vec{\xi}^{\pm}]=\exp\Bigg\{\frac{i}{\hbar}\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'\sum_k\frac{2e^2}{L^3}[\dot{\vec{\xi}}^+(\tau)\dot{\vec{q}}(\tau')G_k^{++}(\tau-\tau')+\dot{\vec{\xi}}^+(\tau)\dot{\vec{q}}(\tau')G_k^{+-}(\tau-\tau')+\dot{\vec{\xi}}^-(\tau)\dot{\vec{q}}(\tau')G_k^{--}(\tau-\tau')+\dot{\vec{\xi}}^-(\tau)\dot{\vec{q}}(\tau')G_k^{-+}(\tau-\tau')]\Bigg\}\exp\Bigg\{\frac{i}{\hbar}\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'\sum_k\frac{2e^2}{L^3}\times[\dot{\vec{\xi}}^+(\tau)\dot{\vec{\xi}}^+(\tau')G_k^{++}(\tau-\tau')+\dot{\vec{\xi}}^+(\tau)\dot{\vec{\xi}}^-(\tau')G_k^{+-}(\tau-\tau')+\dot{\vec{\xi}}^-(\tau)\dot{\vec{\xi}}^-(\tau')G_k^{--}(\tau-\tau')+\dot{\vec{\xi}}^-(\tau)\dot{\vec{\xi}}^+(\tau')G_k^{-+}(\tau-\tau')]\Bigg\}, \quad (3.27)$$

where we used $G_k^{+-}(\tau-\tau')=G_k^{-+}(\tau'-\tau)$ and $G_k^{++}+G_k^{--}+G_k^{-+}+G_k^{+-}=0$. Together with the condition, $\langle\xi^+(t')\rangle=0$ and taking the perturbation theory up to second order, we get the following equation of motion:

$$\int_{-\infty}^{t}d\tau'\langle\vec{\xi}^+(\tau)\dot{\vec{\xi}}^+(\tau')\rangle\Bigg[\Bigg\{m\dot{\vec{q}}(\tau')+\int_{-\infty}^{t}d\tau''\Gamma\times(\tau'-\tau'')\dot{\vec{q}}(\tau'')\Bigg\}+\langle\vec{\xi}^+(\tau)\vec{\xi}^+(\tau')\rangle\vec{j}(\tau')\Bigg]=0, \quad (3.28)$$

where

$$\Gamma(\tau'-\tau'')=\frac{2e^2}{L^3}\sum_k[G_k^{++}(\tau'-\tau'')+G_k^{+-}(\tau'-\tau'')]=\frac{2e^2}{L^3}\sum_k\frac{\sin[\omega_k(\tau'-\tau'')]}{\omega_k}\theta(\tau'-\tau''), \quad (3.29)$$

in accordance with Eq. (3.19).

Integrating out the τ' in Eq. (3.28), we get

$$m\dot{\vec{V}}(\tau)+\int_{-\infty}^{\tau}d\tau'\Sigma(\tau-\tau')\vec{V}(\tau')=\vec{j}(\tau), \quad (3.30)$$

where

$$\Sigma(\tau-\tau')=\frac{\partial}{\partial\tau}\Gamma(\tau-\tau'). \tag{3.31}$$

Making use of Eq. (3.29), we find

$$\Sigma(\tau-\tau')=\frac{2e^2}{L^3}\sum_k\left\{\cos[\omega_k(\tau-\tau')]\theta(\tau-\tau')+\frac{\sin[\omega_k(\tau-\tau')]}{\omega_k}\delta(\tau-\tau')\right\}. \tag{3.32}$$

As a consequence of the use of adiabatic approximation,

$$\int_{-\infty}^{t}dt'\Sigma(t-t')=0, \quad (t<0) \tag{3.33}$$

when we switch on the external classical field at $t=0$, if the charge is with a constant velocity $\vec{v}_0$ for $t<0$, it will be accelerated and will radiate, transfer energy and momentum to the radiated photons, getting to the process of dissipation. So, writing the velocity of charge as $\vec{V}(t)=\vec{v}_0+\vec{v}(t)$, we have

$$m\dot{\vec{v}}(\tau)+\int_0^{\tau}d\tau'\Sigma(\tau-\tau')\vec{v}(\tau')=\vec{j}(\tau) \tag{3.34}$$

considering Eq. (3.33).

The solution for the equation of motion can be found using the Laplace transform of the velocity $\tilde{\vec{v}}(s)$, self-energy of the kernel $\tilde{\Sigma}(s)$ and of the external force $\tilde{\vec{j}}(s)$. In terms of the Laplace variable *s*, we get

$$\tilde{\vec{v}}(s)=\frac{\vec{v}_0+(\tilde{\vec{j}}(s)/m)}{s+\frac{1}{m}\tilde{\Sigma}(s)}. \tag{3.35}$$

The evolution on the real time can be found via inverse Laplace transform

$$\vec{v}(t)=\frac{1}{2\pi i}\int_c e^{st}\tilde{\vec{v}}(s)ds, \tag{3.36}$$

where c is the Bromwich contour, that is at the imaginary axe at the right of all singularities of $\tilde{\vec{v}}(s)$ in the complex plan *s*. For the purpose of understanding the real time dynamics, we need to study the analytical structure of $G(s)\equiv[s+(1/m)\tilde{\Sigma}(s)]^{-1}$. From Eqs. (3.29) and (3.31) we have

$$\tilde{\Sigma}(s)=s\tilde{\Gamma}(s), \tag{3.37}$$

$$\tilde{\Gamma}(s)=\frac{2e^2}{L^3}\sum_k\frac{1}{s^2+\omega_k^2}, \tag{3.38}$$

where $\tilde{\Gamma}(s)$ is the Laplace transform of $\Gamma(\tau-\tau')$.

The presence of static damping coefficient is given by a pole in $G(s)$ with a negative real part, because this turns in an exponential of velocity relaxation, as occurs in the damping harmonic oscillator.

In the absence of interaction, $G(s)$ has a simple pole at $s=0$, that is, the static damping coefficient cancels, $[\tilde{\Sigma}(s)=0]$ for $(t\to\infty)$, meaning that the system does not present radiation damping. This is consistent with the discussions about the accelerated electron [11], where it is shown the effect of radiation damping is relevant only for short time intervals ($\sim10^{-24}$ s for electron).

B. Semiclassical Langevin equation

Another important method used to study the nonequilibrium dynamics of a particle coupled to a dissipative medium is finding the equation of motion that looks like the Langevin equation for a particle taking in account the effect of the medium. This interaction introduces a stochastic dissipation term in the equation of motion, both related by the fluctuation-dissipation theorem.

A description of the dynamic of nonequilibrium of the system by the Langevin equation can be seen in many papers, e.g, [3,4], among others. In these papers, the starting point is the application of the Feynman and Vernon formalism [1] for the generating functional, what gives naturally the semiclassical Langevin equation.

The equation (3.23) is the first step that was already done. Now, considering new coordinates, $\vec{r}$ and $\vec{R}$, defined as

$$\vec{r}(t)=\frac{1}{2}[\vec{x}^+(t)+\vec{x}^-(t)], \tag{3.39}$$

$$\vec{R}(t)=\vec{x}^+(t)-\vec{x}^-(t), \tag{3.40}$$

we find

$$\tilde{Z}[j]=\int\mathcal{D}\vec{r}(t)\mathcal{D}\vec{R}(t)e^{i/\hbar\tilde{S}[\vec{r},\vec{R}]}\mathcal{F}[\vec{r},\vec{R}] \tag{3.41}$$

with

$$\tilde{S}[\vec{r},\vec{R}]=\int_{-\infty}^{t}\{m\dot{\vec{R}}(\tau)\dot{\vec{r}}(\tau)-\vec{j}(\tau)\vec{R}(\tau)\}d\tau \tag{3.42}$$

and

$$\mathcal{F}[\vec{r},\vec{R}]=\exp\Bigg\{\frac{i}{\hbar}\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'\sum_k\frac{e^2}{L^3}\times\Bigg[\frac{\dot{\vec{R}}(\tau)\dot{\vec{R}}(\tau')}{4}\{G_k^{++}(\tau)+G_k^{--}(\tau)-G_k^{+-}(\tau)-G_k^{-+}(\tau)\}+\frac{\dot{\vec{R}}(\tau)\dot{\vec{r}}(\tau')}{2}\{G_k^{++}(\tau)-G_k^{--}(\tau)+G_k^{+-}(\tau)-G_k^{-+}(\tau)\}+\frac{\dot{\vec{r}}(\tau)\dot{\vec{R}}(\tau')}{2}\{G_k^{++}(\tau)-G_k^{--}(\tau)-G_k^{+-}(\tau)+G_k^{-+}(\tau)\}\Bigg]\Bigg\}, \tag{3.43}$$

where $G_k^{ab}(\tau)=G_k^{ab}(\tau-\tau')$. With the change $\tau\leftrightarrow\tau'$ in the third term of $\mathcal{F}[\vec{r},\vec{R}]$ and with Eq. (3.19), the influence functional is

$$\mathcal{F}[\vec{r},\vec{R}]=\exp\Bigg\{\frac{i}{\hbar}\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'\Bigg[\dot{\vec{R}}(\tau)\gamma_I(\tau-\tau')\dot{\vec{r}}(\tau')+\frac{i}{2}\dot{\vec{R}}(\tau)\gamma(\tau-\tau')\dot{\vec{R}}(\tau')\Bigg]\Bigg\} \tag{3.44}$$

with

$$\gamma_I(\tau-\tau')=\frac{2e^2}{L^3}\sum_k\frac{\sin[\omega_k(\tau-\tau')]}{\omega_k}\theta(\tau-\tau') \tag{3.45}$$

and

$$\gamma(\tau-\tau')=\frac{e^2}{L^3}\sum_k\frac{\cos[\omega_k(\tau-\tau')]}{\omega_k}\coth\left(\frac{\beta\hbar\omega_k}{2}\right), \tag{3.46}$$

and after some integrations we get

$$\mathcal{F}[\vec{r},\vec{R}]=\exp\Bigg\{-\frac{i}{\hbar}\int_{-\infty}^{t}d\tau\int_{-\infty}^{t}d\tau'\Bigg[\vec{R}(\tau)K_I(\tau-\tau')\dot{\vec{r}}(\tau')-\frac{i}{2}\vec{R}(\tau)K(\tau-\tau')\vec{R}(\tau')\Bigg]\Bigg\}, \tag{3.47}$$

where

$$K_I(\tau-\tau')=\frac{\partial}{\partial\tau}\gamma_I(\tau-\tau'), \tag{3.48}$$

$$K(\tau-\tau')=-\frac{\partial^2}{\partial\tau^2}\gamma(\tau-\tau'). \tag{3.49}$$

In terms of the new variables defined in Eqs. (3.39) and (3.40), we see that the influence functional (3.44) has exactly the same form as the one obtained in the quantum Brownian motion [24], except that the γ_I coefficient presents memory, i.e., it is non-Markovian. Then we get

$$\tilde{Z}[j]=\int\mathcal{D}\vec{r}(t)\mathcal{D}\vec{R}(t)e^{(i/\hbar)S[\vec{r},\vec{R}]} \tag{3.50}$$

with the nonequilibrium effective action

$$S[\vec{r},\vec{R}]=\int d\tau\vec{R}(\tau)\Bigg[-m\ddot{\vec{r}}(\tau)-\int d\tau' K_I(\tau-\tau')\dot{\vec{r}}(\tau')+\frac{i}{2}\int d\tau' K(\tau-\tau')\vec{R}(\tau')-\vec{j}(\tau)\Bigg], \tag{3.51}$$

where $K_I(\tau-\tau')$ and $K(\tau-\tau')$, in accordance with Eqs. (3.45) and (3.46), is given by

$$K_I(\tau-\tau')=\frac{2e^2}{L^3}\sum_k\left\{\cos[\omega_k(\tau-\tau')\theta(\tau-\tau')]+\frac{\sin[\omega_k(\tau-\tau')]}{\omega_k}\delta(\tau-\tau')\right\}=\Sigma(\tau-\tau') \tag{3.52}$$

and the second term cancels in Eq. (3.51), and

$$K(\tau-\tau')=\frac{e^2}{L^3}\sum_k\omega_k\cos[\omega_k(\tau-\tau')]\coth\left(\frac{\beta\hbar\omega_k}{2}\right). \tag{3.53}$$

The integral in Eq. (3.50) can be written as

$$\exp\left[-\frac{1}{2\hbar}\int d\tau\int d\tau'\vec{R}(\tau)K(\tau-\tau')\vec{R}(\tau')\right]=C(t)\int\mathcal{D}\zeta\exp\left[-\frac{\hbar}{2}\int d\tau\int d\tau'\vec{\zeta}(\tau)K^{-1}\times(\tau-\tau')\vec{\zeta}(\tau')+\frac{i}{\hbar}\int d\tau\vec{\zeta}(\tau)\vec{R}(\tau)\right], \tag{3.54}$$

where $C(t)$ is a multiplicative factor, and so the generating functional becomes

$$\tilde{Z}[j]=\int\mathcal{D}\vec{r}(\tau)\mathcal{D}\vec{R}(\tau)\mathcal{D}\vec{\zeta}(\tau)P[\vec{\zeta}]\exp\Bigg\{\frac{i}{\hbar}\int d\tau\vec{R}(\tau)\times\Bigg[-m\ddot{\vec{r}}(\tau)-\int d\tau' K_I(\tau-\tau')\dot{\vec{r}}(\tau')+\vec{\zeta}(\tau)-\vec{j}(\tau)\Bigg]\Bigg\} \tag{3.55}$$

with the probabilistic distribution of stochastic noise $P[\vec{\zeta}]$ given by

$$P[\vec{\zeta}]=\int\mathcal{D}\vec{\zeta}\exp\left\{-\frac{\hbar}{2}\int d\tau\int d\tau'\vec{\zeta}(\tau)K^{-1}(\tau-\tau')\vec{\zeta}(\tau')\right\}, \tag{3.56}$$

where ζ represents fluctuation force of the bath on short time scales with the Gaussian-noise correlation function

$$\langle\vec{\zeta}(\tau)\vec{\zeta}(\tau')\rangle=\hbar K(\tau-\tau'). \tag{3.57}$$

The semiclassical Langevin equation is obtained extremizing the action in Eq. (3.55) in relation to $\vec{R}(\tau)$,

$$m\dddot{\vec{r}}(\tau)+\int_{-\infty}^{\tau}d\tau'\Sigma(\tau-\tau')\dot{\vec{r}}(\tau')+\vec{j}(\tau)=\vec{\zeta}(\tau). \tag{3.58}$$

This is a typical Langevin equation, but that is not the only evolution equation we can get, since we can derive with respect to $\vec{r}(\tau)$ and to the noise term $\vec{\zeta}(\tau)$.

We can see from Eqs. (3.52) and (3.53) that $K_I(\tau-\tau')$ is non-Markovian, that is, it presents memory and the correlation function $K(\tau-\tau')$, but is not a δ function like in the quantum Brownian motion [24]. As consequence of this fact, the correlation function of the noise, under some conditions, can be written as a correlation function of classical forces

$$\langle\vec{\zeta}(\tau)\vec{\zeta}(\tau')\rangle\equiv 2\eta kT\delta(\tau-\tau'), \tag{3.59}$$

in the classical limit of $kT\gg\hbar\omega_k$, shown by Caldeira and Leggett [24]. Finaly, we see that taking the average of Eq. (3.58) with the probality distribution $P[\vec{\zeta}]$ we get the equation of motion (3.30) in the particle coordinate.

We have obtained the same equation of motion by this different approach, giving us some confidence in its validity and generality, and in our conclusion we have obtained a dissipative term arising from the self-field (self-correlation) that corresponds to the classical counterpart.

IV. CONCLUSION

The generating functional for an accelerated charge is computed taking a partial trace over the photons degrees of freedom without any approximation obtaining the Green's function for the charge. An important feature for this generating function is the equivalence to the harmonic oscillator with sources.

Making an approximation in the modes of the field, we get the equation of motion in real time for the charge that presents a self-energy kernel that is non-Markovian and a static damping coefficient that exists only for short intervals of time.

We analyze the resulting Langevin equation and show that it has the same form as the one for the quantum Brownian motion, but with a non-Markovian damping coefficient that presents no dependence with the field temperature and a Gaussian noise.

We have not yet attempted to reproduce the Abraham-Becker/Hartemann-Luhmann equations in the classical limit.

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APPENDIX: CONTOUR PATH INTEGRALS IN REAL TIME

In this section, we are going to present a powerful method that is called the closed-time-path (CTP) functional-integral formalism and provides the technical means to study the nonequilibrium dynamics of the quantum field theory in a causal manner. This formalism was developed by Schwinger, Keldysh, Korenman, and others [9,14–18,10,19–21].

The density operator ρ contains all the information about the ensemble. For a system described by a Hamiltonian H, the density operator satisfies the quantum Liouville equation

$$i\hbar\frac{\partial\rho(t)}{\partial t}=[H(t),\rho(t)]. \tag{A1}$$

In equilibrium statistical mechanics, the density operator is independent of time which means that the density operator commutes with H. However, in nonequilibrium statistical mechanics the density operator does depend on time and the goal of nonequilibrium quantum field theory is to study the time evolution of the density operator.

The formal solution of Liouville equation is

$$\rho(t)=U(t,t_i)\rho(t_i)U^{-1}(t,t_i), \tag{A2}$$

where $\rho(t_i)$ is the initial density operator at time t_i that determines the initial conditions for the evolution and $U(t,t_i)$ is the time development operator defined as

$$U(t_f,t_i)=\exp\left[-\frac{i}{\hbar}\int_{t_i}^{t_f}H_s(t')dt'\right]. \tag{A3}$$

The case that is particularly of interest to us is the quantum evolution of closed system in a time dependent background. Let us consider the case where the Hamiltonian is time dependent after some time say t_i (for example, an interaction is switched on after t_i) and it is time independent before that time, i.e., $H(t)=H_i$ for $t\leq t_i$. This means that we assume that the system has been in equilibrium up to t_i and will evolve out of equilibrium thereafter. In this case, the initial density operator is given by

$$\rho(t_i)=e^{-\beta H_i}\quad\text{for}\ \ t\leq t_i \tag{A4}$$

where $\beta=1/T$ and T is the temperature of the system.

The expectation value of any operator O is

$$\langle O(t)\rangle=\frac{\mathrm{Tr}[U(t,t_i)e^{-\beta H_i}U^{-1}(t,t_i)O]}{\mathrm{Tr}[e^{-\beta H_i}]}=\frac{\mathrm{Tr}[e^{-\beta H_i}U(t,t_i)OU^{-1}(t,t_i)]}{\mathrm{Tr}[e^{-\beta H_i}]}, \tag{A5}$$

where we used the trace property $\mathrm{Tr}(ABC)=\mathrm{Tr}(BCA)$ and $U^{-1}(t,t_i)=U(t,t_i)$ in the second equality.

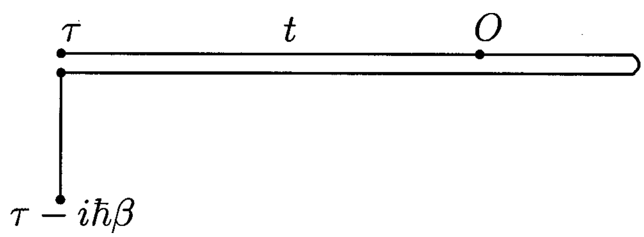

FIG. 1. Real-time contours.

Now consider a time $\tau < t_i$ where $H(t) = H_i$ is independent of time. Then the time development operator is given by

$$U(\tau,t_i) = \exp\left[-\frac{i}{\hbar} H_i(\tau - t_i) \right], \tag{A6}$$

which commutes with $e^{-\beta H_i}$. The density operator can be expressed in terms of the time development operator with no sources by writing

$$e^{-\beta H_i} = \exp\left[-\frac{i}{\hbar} H_i(\tau - i\hbar\beta - \tau) \right] = U(\tau - i\hbar\beta, \tau). \tag{A7}$$

Thus Eq. (A5) can be written as

$$\langle O(t) \rangle = \frac{\mathrm{Tr}[U(\tau - i\hbar\beta,\tau)U(t_i,\tau)U(\tau,t_i)U(t_i,t)OU(t,t_i)]}{\mathrm{Tr}[e^{-\beta H_i}]} \tag{A8}$$

where $U(t_i,\tau)U(\tau,t_i) = 1$ was inserted in the trace. Commuting $U(t_i,\tau)$ with $\rho(t_i)$ and using the composition property, $U(t_i,t_i) = 1$ *e* $U(t_f,t_1)U(t_1,t_i) = U(t_f,t_i)$, one obtains

$$\langle O(t) \rangle = \frac{\mathrm{Tr}[U(\tau - i\hbar\beta,\tau)OU(t,\tau)]}{\mathrm{Tr}[e^{-\beta H_i}]}. \tag{A9}$$

This corresponds to a process starting at time $\tau < t_i$, propagating to time t, inserting O, and propagating back from time t to $\tau - i\hbar\beta$ in complex time plane, see Fig. 1. The generalization to real-time correlation functions of Heisenberg picture is straightforward. Correlation functions of operators may be obtained from the generating functional that couples the operators to sources, taking functional derivatives with respect to sources and then setting the sources to zero, in such a way that we can define the nonequilibrium generating functional as

$$Z[J^+,J^-] = \frac{\mathrm{Tr}[U_{J^+}(t,\tau)U_{J^\beta}(\tau - i\hbar\beta,\tau)U_{J^-}^{-1}(t,\tau)]}{\mathrm{Tr}[\rho(t_i)]}, \tag{A10}$$

where

$$U_J(t_i,t_f) = T\exp\left\{ -\frac{i}{\hbar}\int_{t_i}^{t_f} dt'(H(t') + J(t')q(t')) \right\}, \tag{A11}$$

with the sources $J = J^+$ being the source in the upper part of the contour, segment $\tau \to t$, $J = J^-$ in the lower part, segment $t \to \tau$, and $J = J^\beta$ in the imaginary part, segment $\tau \to \tau - i\hbar\beta$. The trace in Eq. (A10) may be calculated by introducing a complete set of coordinate(or field in quantum field theory) eigenstates $\hat{q}(t)|q\rangle = q(t)|q\rangle$. Doing so, we have

$$Z[J^+,J^-,J^\beta] = \int \mathcal{D}q\mathcal{D}q_1\mathcal{D}q_2 \langle q|U_{J^\beta}(\tau - i\hbar\beta,\tau)|q_1\rangle \times \langle q_1|U_{J^-}(t,\tau)|q_2\rangle\langle q_2|U_{J^+}(\tau,t)|q\rangle. \tag{A12}$$

Using the result (see any textbook on path integral)

$$\langle q_b|U_J(t_b,t_a)|q_a\rangle = \int_{q(t_{a(b)}) = q_{a(b)}} \mathcal{D}q \exp\left\{ \frac{i}{\hbar}\int_{t_a}^{t_b} dt \mathcal{L}[q,J] \right\} \tag{A13}$$

we get the generating functional written as

$$Z[J^+,J^-,J^\beta] = \int \mathcal{D}q\mathcal{D}q_1\mathcal{D}q_2 \int \mathcal{D}q^+\mathcal{D}q^-\mathcal{D}q^\beta \times \exp\left\{ \frac{i}{\hbar}\int_\tau^t (\mathcal{L}[q^+,J^+] - \mathcal{L}[q^-,J^-])dt \right\} \times \exp\left\{ \frac{i}{\hbar}\int_\tau^{\tau - i\hbar\beta} \mathcal{L}[q^\beta,J^\beta]dt \right\}, \tag{A14}$$

where $\mathcal{L}[q,J] = \mathcal{L}[q] + \hbar Jq$, with the boundary conditions $q^+(\tau) = q^\beta(\tau - i\hbar\beta) = q$; $q^+(t) = q^-(\tau) = q_2$ and $q^-(\tau) = q^\beta(\tau) = q_1$.

Having established the path-integral representation for the generating functional, we now rewrite it in the interaction picture, with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$. In the usual manner [29,30] we replace the coordinate in the interaction Lagrangian, $\mathcal{L}_{int}(q)$, by a functional differentiation with respect to the source functions to obtain

$$Z[J^+,J^-,J^\beta] = \exp\left\{ \frac{i}{\hbar}\int_\tau^t [\mathcal{L}_{int}(-i\hbar\delta/\delta J^+) - \mathcal{L}_{int}(i\hbar\delta/\delta J^-)]dt \right\} \times \exp\left\{ \frac{i}{\hbar}\int_\tau^{\tau - i\hbar\beta} \mathcal{L}_{int}(-i\hbar\delta/\delta J^\beta)dt \right\} \times Z_0[J^+,J^-,J^\beta] \tag{A15}$$

such that Z_0 is the same Z but with $\mathcal{L}$ replaced by the noninteracting $\mathcal{L}_0$.

Integrating the quadratic terms in Z_0, the generating functional can be cast in the following form:

$$Z[J^+,J^-,J^\beta]=\exp\left\{\frac{i}{\hbar}\int_\tau^t[\mathcal{L}_{int}(-i\hbar\,\delta/\delta J^+)-\mathcal{L}_{int}(i\hbar\,\delta/\delta J^-)]dt\right\}\times\exp\left\{\frac{i}{\hbar}\int_\tau^{\tau-i\hbar\beta}\mathcal{L}_{int}(-i\hbar\,\delta/\delta J^\beta)dt\right\}\times\exp\left\{\frac{i}{2\hbar}\int_c dt_1\int_c dt_2 J_c(t_1)J_c(t_2)\times G_c(t_1,t_2)\right\}. \tag{A16}$$

Taking the limit $\tau\to\infty$, the crossed terms in Z vanish as consequence of the Riemann-Lesbegue's lemma [19], then

$$Z[J^+,J^-,J^\beta]=\exp\left\{\frac{i}{\hbar}\int_\tau^t[\mathcal{L}_{int}(-i\hbar\,\delta/\delta J^+)-\mathcal{L}_{int}(i\hbar\,\delta/\delta J^-)]dt\right\}\times\exp\left\{\frac{i}{\hbar}\int_\tau^{\tau-i\hbar\beta}\mathcal{L}_{int}(-i\hbar\,\delta/\delta J^\beta)dt\right\}\times\exp\left\{\frac{i}{2\hbar}\int_\tau^{\tau-i\hbar\beta}d\tau_1\int_\tau^{\tau-i\hbar\beta}d\tau_2 J_\beta(\tau_1)\times G_\beta(\tau_1-\tau_2)J_\beta(\tau_2)\right\}\times\exp\left\{\frac{i}{2\hbar}\int_\tau^t dt'\int_\tau^t dt'' J_a(t')\times G_{ab}(t'-t'')J_b(t'')\right\} \tag{A17}$$

with t',t'' the real time variable and τ_1,τ_2 running down the imaginary time axis (Fig. 1). This term cancels between denominator and numerator in correlation functions. Here $a,b=+,-$, and finally we are led to the generating functional of *real-time* correlation functions at finite temperature, in or out of equlibrium

$$Z[J^+,J^-]=\exp\left\{\frac{i}{\hbar}\int_\infty^{-\infty}[\mathcal{L}_{int}(-i\hbar\,\delta/\delta J^+)-\mathcal{L}_{int}(i\hbar\,\delta/\delta J^-)]dt\right\}\times\exp\left\{\frac{i}{2\hbar}\int_{-\infty}^\infty dt'\int_{-\infty}^\infty dt'' J_a(t')\times G_{ab}(t'-t'')J_b(t'')\right\}, \tag{A18}$$

where the Green's functions G_{ab} are given by

$$\begin{aligned}G^{++}(\vec r_1,t_1;\vec r_2,t_2)&=G^>(\vec r_1,t_1;\vec r_2,t_2)\,\theta(t_1-t_2)+G^<(\vec r_1,t_1;\vec r_2,t_2)\,\theta(t_2-t_1),\\ G^{--}(\vec r_1,t_1;\vec r_2,t_2)&=G^>(\vec r_1,t_1;\vec r_2,t_2)\,\theta(t_2-t_1)+G^<(\vec r_1,t_1;\vec r_2,t_2)\,\theta(t_1-t_2),\\ G^{+-}(\vec r_1,t_1;\vec r_2,t_2)&=-G^<(\vec r_1,t_1;\vec r_2,t_2),\\ G^{-+}(\vec r_1,t_1;\vec r_2,t_2)&=-G^>(\vec r_1,t_1;\vec r_2,t_2)=-G^<(\vec r_2,t_2;\vec r_1,t_1).\end{aligned} \tag{A19}$$

These Green's functions satisfy the following relation:

$$G_k^{++}+G_k^{--}+G_k^{-+}+G_k^{+-}=0. \tag{A20}$$

In the case of radiation damping, in which the generating functional (3.15) is equivalent to the harmonic oscillator we get

$$G_k^<(\tau-\tau')=\frac{i}{2\omega_k}[e^{i\omega_k(\tau-\tau')}(1+\eta_k)+e^{-i\omega_k(\tau-\tau')}\eta_k], \tag{A21}$$

$$G_k^>(\tau-\tau')=\frac{i}{2\omega_k}[e^{-i\omega_k(\tau-\tau')}(1+\eta_k)+e^{i\omega_k(\tau-\tau')}\eta_k], \tag{A22}$$

where $\eta_k=(e^{\beta\hbar\omega_k}-1)^{-1}$ is the ocupation number.

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